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PRINCIPAL MECHANISM OF MICRO-LIQUID-LAYER FORMATION ON A SOLID SURFACE WITH A GROWING BUBBLE IN NUCLEATE BOILING

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Abstract— Relating to nucleate boiling, mechanism of micro-liquid-layer formation between a solid surface and a growing bubble is investigated. In order to approach the essential nature of the subject, a flattened air bubble is generated in a narrow space between a pair of parallel disks filled with a liquid, and the behavior of the liquid-layer formed on the solid surface is observed by optical means.

According to the experimental results, the formation of micro-liquid-layer is nearly regarded as a local phenomenon related to the flow of liquid near the front meniscus of bubble; and an empirical equation is derived in respect of the profile of micro-liquid-layer.

Theoretical analysis is attempted to study the flow of liquid-layer in which surface tension plays an important role. It provides a theoretical ground for the empirical equation aforementioned. Experimental data of micro-liquid-layer obtained in nucleate boiling of water in a narrow space between a pair of parallel disks are also used to examine the reliability of the empirical equation.

Summarizing the study, it is concluded that a liquid-layer put between a bubble and a solid surface is subject to positional change of pressure due to the effect of surface tension, thereby the liquid-layer is driven to flow against viscous force, leaving a micro-liquid-layer with thickness of the order of microns behind.

NOMENCLATURE

- C, a numerical constant;
- H, half of normal distance between a pair of parallel discs;
- h, thickness of liquid-layer (height of liquid-layer measured from a solid surface);
- K, a numerical constant;
- k, a numerical constant;
- m, a numerical constant;
- n, a numerical constant;
- p, pressure in air chamber (gauge pressure);
- r, radial coordinate (distance measured from the center of a disc);
- t, time;
- y, vertical coordinate (distance measured from a disc surface);
- U_0, spreading velocity of a bubble;
- U, horizontal component of velocity of gas–liquid interface;
- u, horizontal component of velocity of liquid;
- V, $\equiv \int_0^h u\,dy$.

Greek symbols

- ζ, dimensionless thickness of liquid-layer, defined in equation (7);
- η, dimensionless radial coordinate, defined in equation (7);
- μ, viscosity of liquid;
- ρ, curvature of gas–liquid interface (ρ is negative when the center of curvature is in the side of liquid);
- σ, surface tension.

1. INTRODUCTION

Not a few reports have already been published suggesting the existence of micro-liquid-layer between a solid surface and a growing bubble in nucleate boiling. Moore and Mesler [1] observed rapid temperature drop of a heated surface subject to boiling, and they presumed existence of a micro-liquid-layer and its evaporation. Then, Rogers and Mesler [2], Hendricks and Sharp [3] and Cooper and Lloyd [4] supported it through their simultaneous measurements of the bubble growth and temperature drop of heated surface. Meanwhile, Sharp [5], Torikai and Yamazaki [6] observed the micro-liquid-layer positively by optical means, and Hospetti and Mesler [7] attempted to observe the characteristics of scale put at active nucleation sites in nucleate boiling of water saturated with radio-active calcium-sulfate. For a special case of nucleate boiling in a narrow space between a pair of parallel disks, Katto and Yokoya [8] reported that an isolated bubble spread radially in the narrow space creating a micro-liquid-layer on the heated surface and the evaporation of the liquid-layer dominated the bubble growth as well as heat transfer.

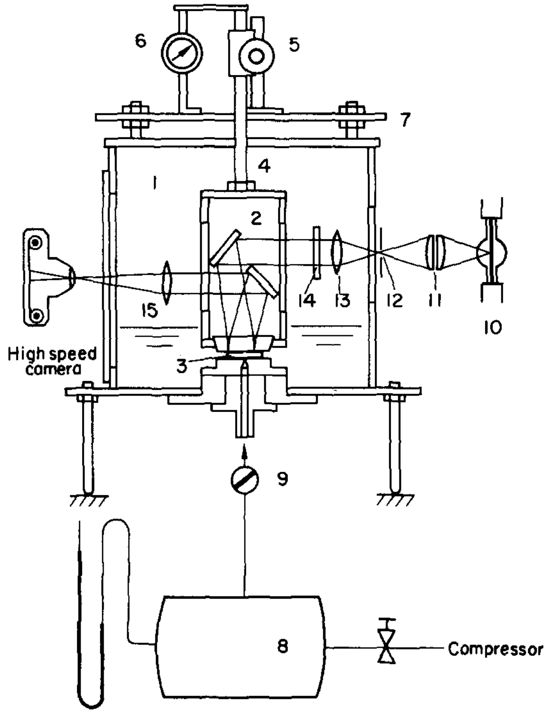
Of course, it should be mentioned that the role played by such a micro-liquid-layer has not yet been clarified for the ordinary nucleate boiling. In order to make it clear, it is necessary to accumulate positive informations more and more; and this problem will be reserved in the present study. It is an actual fact at least, however, that there are several undoubted cases in which a micro-liquid-layer is created between a heated surface and a growing bubble; and accordingly, how the micro-liquid-layer is built up is a problem of great importance. As to this problem, Torikai [9], Kotake [10] and Cooper [11] have already presented theoretical analysis or tentative presumption. However, they have a common idea regarding the phenomenon as concerned with the flow of liquid within the conventional range of hydrodynamics in spite of an individual interpretation put respectively in their studies; and noticing that the liquid-layer is extraordinarily thin of the order of microns, it seems likely doubtful whether such a way of attack is reasonable or not. In addition, since the pressure should be approximately uniform within a micro-liquid-layer exposed to a vapor bubble if the effect of surface tension is omitted, and since viscosity of vapor is negligibly small in general compared with that of liquid, it is quite unreasonable to assume either the pressure difference within the liquid-layer or the shearing stress on the vapor–liquid interface as a driving force of flow of liquid.

With such a point of view in mind, and in order to clarify straightforwardly the essential nature of the phenomenon, the present study utilizes radial spreading of a flattened air bubble which is produced artificially within a narrow space between a pair of parallel discs filled with a transparent liquid, and the behavior of the micro-liquid-layer thus created on a solid surface is observed. Theoretical examination is also attempted, and the authors intend to clarify the principal mechanism with which a micro-liquid-layer is formed on a solid surface with a growing bubble.

2. EXPERIMENTAL APPARATUS

Experimental apparatus is shown schematically in Fig. 1. Main vessel ①, in which a transparent liquid is kept in pool (water and methanol is used respectively in the present study), holds an optical box ② as well as a disk ③ in coaxial position. An optical flat of 30 mm dia., which is set up at the bottom of the optical box ②, co-operates with the disk ③ of the same diameter as the optical flat in setting up a narrow space between them. A fine hole of 0·5 mm dia. has been bored at the center of upper surface of disk ③ for air-injection, and a flattened air bubble is forced to grow radially in the narrow space removing liquid and generating a thin liquid-layer on the upper and lower solid surfaces respectively. The growth of air bubble and the thickness of liquid-layer formed on the lower surface of the optical flat are measured simultaneously by means of the light-interference method as well as high-speed photography. Since the immediate problem is a matter of hydrodynamics, heating is not required.

Height of the optical box ② measured from the upper surface of the disk ③, that is, the distance between the two parallel planes of the space in which an air bubble grows, can be nut, so that the parallelness between the optical flat and disk ③ also can be easily regulated by adjusting the relative inclination of the levelling disk ⑦.



1	Main vessel	6	Dial gauge	11	Condensing lens
2	Optical box	7	Levelling disk	12	Pin-hole
3	Disk	8	Air chamber	13	Collimating lens
4	Supporting bar	9	Cock	14	Interference filter
5	Fine adjustment screw	10	Source of light	15	View adjustment lens

FIG. 1. Experimental apparatus.

adjusted by means of a fine adjustment screw ⑤ which is related to the optical box through a supporting bar ④; and the displacement of the optical box is measured with sufficient accuracy by a dial gauge ⑥. The afore-mentioned equipments of ⑤ and ⑥ are fixed upon a levelling disk ⑦ which is related to the main vessel ① in three-legged form through three sets of bolt and

In the experiment, air which has been stored in an air chamber ⑧ of 300 cm^3 in volume with a desired pressure, p, is supplied by opening a cock ⑨ abruptly to make a rapidly growing air bubble. As the initial pressure of the air chamber is increased, growing velocity of air bubble becomes greater of course.

For a source of light ⑩, super-high-pressure mercury arc of d.c. type and of 250 W is used not only to keep the intensity of light required in the high-speed photography but also to avoid the cyclic change of intensity of light. The light supplied from the source is collected on a pin-hole ⑫ of 1 mm in diameter with a condenser ⑪ and then collimated with a collimating lens ⑬. Strictly speaking, somewhat expanding rays of light are used instead of parallel light in the present experiment in order to enlarge the measurable field of vision, but it can be verified from a theoretical examination that the effect arisen from using such rays of light is negligibly small in the present system. After passing through an interference-filter ⑭ consisted of multiple layers of non-metallic material, in which the absorption of light is very little, monocromatic light of 5461 Å in wave length appears and then enters the optical box. Interference fringes made on the lower surface of the optical flat are photographed by a high-speed camera through a view adjustment lens ⑮.

As is shown in Fig. 1, the turn of pathway of light in the optical box is conducted with a pair of mirrors silvered at front surface respectively. It should be mentioned here that the preliminary experiment has shown that the conventional method using two glass prisms is unsuitable due to the occurrence of great loss in the effective intensity of light. Of course, light cannot fall on the surface of the optical flat perpendicularly in the system of a pair of mirrors, but it does not raise a serious problem. The incident angle of 85° is adopted in the present study, for which only a negligibly small error less than 0·4 per cent appears for the measurement of interference fringes.

If the upper and lower surfaces of the optical flat are exactly parallel with each other, the light reflected at the upper surface also enters the high-speed camera and deteriorates the contrast of picture of interference fringes. In order to avoid this obstruction, the upper surface of the optical flat has been designed to have a slight inclination of 1·2° as against the lower surface (an optical flat with coated upper surface for the prevention of the reflection of light was tested in the preliminary experiment, but its performance was considerably inferior than the inclined upper surface). On the other hand, the light reflected at the upper surface of the disk ③ also gives an obstruction to the experiment. Since the light falls also on the surface of the disk ③ with great intensity, it is undesirable to make the disk ③ of such material of high reflexibility as metal. In the present study, a disk made of transparent acrylic resin and put with a black rubber plate as a backing, is used in order to darken the background of field of vision.

3. EXPERIMENT

Liquid-layers treated in the present experiment are extraordinarily thin, so that it is necessary to pay attention to many points in connection with the formation of liquid-layer so as not to induce the disturbance, irregularity, nonuniformity and so on.

Since removing of dusts as well as keeping sufficient affinity with the liquid are quite important for the surface of the optical flat, the surface is washed and cleaned very carefully using distilled water and methanol. The surface of the disk made of acrylic resin, which faces the optical flat, is finished with an emery-paper taking care of the roughness as well as the waviness. In the present study, normal distance of the space in which an air bubble grows is always held at 0·5 mm for the sake of convenience.

Gushing out of the fine nozzle of 0·5 mm in diameter at the center of the disk made of acrylic resin, air has to flow striking against the surface of the optical flat within a certain range at least. Therefore, the liquid-layer made on the surface of the optical flat ought to have special features in the neighborhood of the center. This is confirmed experimentally of course, and the pictures of interference fringes taken at the stage of full development of air bubble are shown in Fig. 2 as a couple of examples: the case of water in Fig. 2(a) and the case of methanol in Fig. 2(b). This nasty phenomenon yields, however, no obstacle to the aim of the present

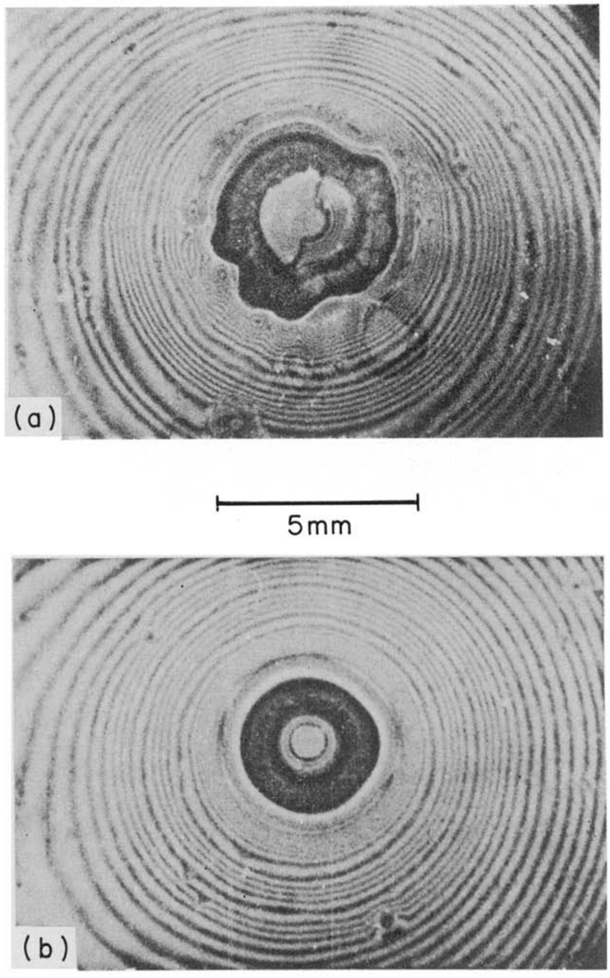


FIG. 2 Interference fringes. (a) Case of water.
(b) Case of methanol.

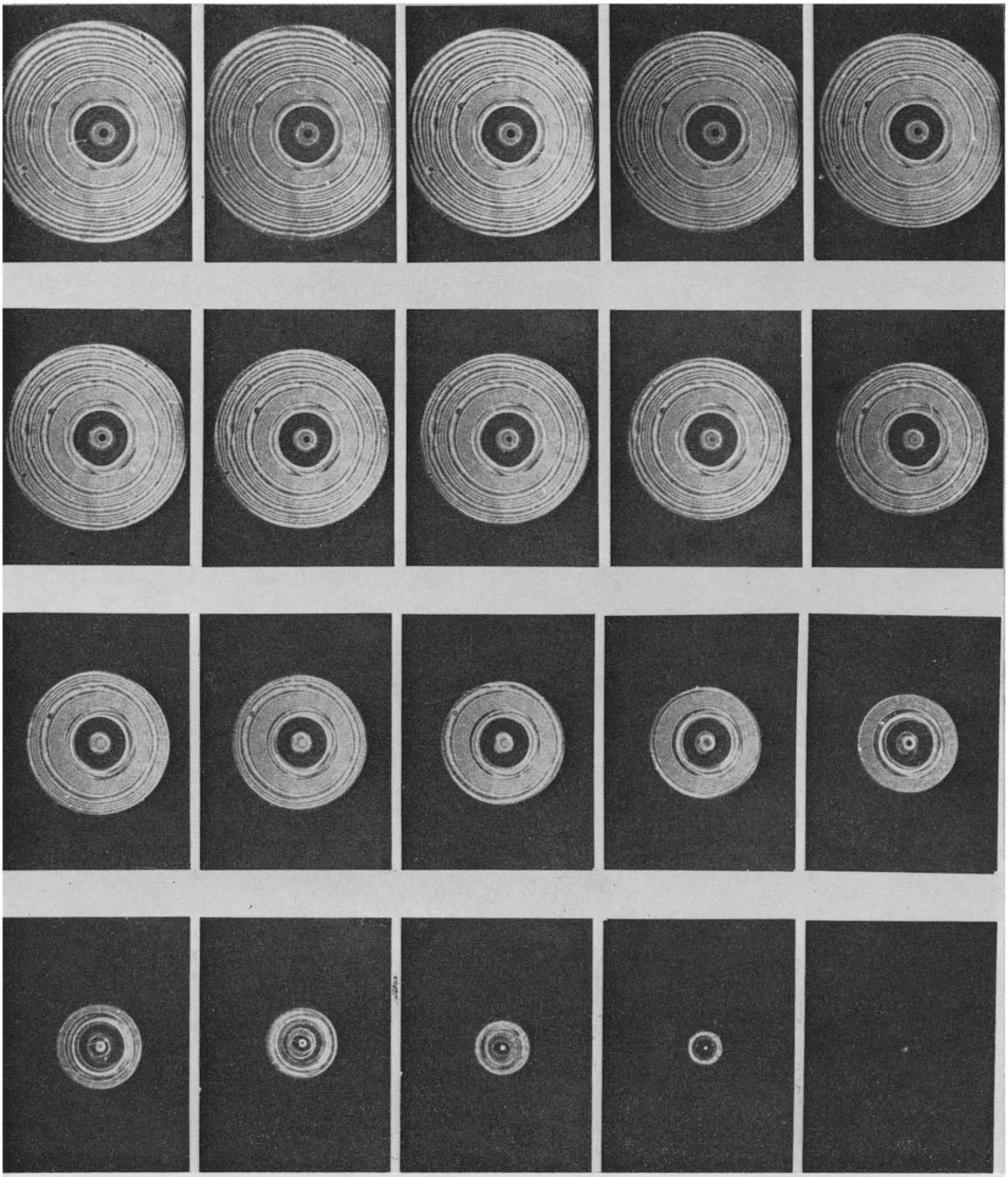


FIG. 3. Successive state of liquid-layer following growth of a flattened bubble.

study as will be clarified in the next chapter; and likewise, the diameter and roundness at the exit of air nozzle does not become dominant factors so far as the present study is concerned (air nozzles of 0·3, 0·5, 0·7 and 2·0 mm dia. with and without roundness at their exit have been tested respectively in the preliminary experiments). As for the growth of air bubble, however, it is necessary of course to pay attention to keep the axially symmetrical flow of air radiating from the air nozzle.

An experimental result with respect to the successive state of a liquid-layer (methanol, at intervals of 2·84 ms) is shown in Fig. 3 as a typical example. After the start of air injection through the air nozzle, a flattened air bubble spreads over the surface of the optical flat generating a round liquid-layer under the bubble. From such a high-speed photographic record as aforementioned, local spreading velocity of a bubble, that is, local moving velocity of the front meniscus of bubble, U_0, can also be determined. Strictly speaking, the front meniscus of the bubble is approximately semicircular in shape, so that such photographs as Fig. 3 taken with parallel rays of light should have some ambiguity regarding the border of bubble. However, since the thickness of the flattened air bubble is only 0·5 mm in the present study, the error induced to the measurement of U_0 is negligibly small. Local value of U_0 varies continuously as a bubble grows, but the variation of U_0 gives no obstacle to the aim of the present study as will be clarified in the next chapter.

Although U_0 can be made greater by raising the initial pressure of the air chamber, p, of course, it should be remembered that disturbance is apt to appear within a liquid-layer if U_0 is raised excessively. In the present study, therefore, measurements are carried out within the range of p = 120–18 mmHg (gauge pressure) and U_0 = 0·75–0·05 m/s. It may not be useless to mention here that the growing velocity of bubbles in the ordinary nucleate boiling is almost in the same order as the aforementioned value of U_0.

4. EXPERIMENTAL RESULTS

4.1 *General remarks*

Examining the experimental results such as shown in Fig. 3, important features can be immediately found as to the phenomenon concerned. First, the gas–liquid interface near the front meniscus of bubble moves rapidly with velocity of U_0, whereas the micro-liquid-layer which is formed continuously behind the moving front meniscus of bubble remains stationary (movement of the micro-liquid-layer is not observable within a period of measurement at least). This suggests that the gas–liquid interface may be approximately divided into two discrete regions: the moving region near the front meniscus of bubble, and the stationary region behind the former. The micro-liquid-layer in the stationary region increases its thickness with the radial distance from the center. In addition, it should be mentioned that even if considerable irregularity appears near

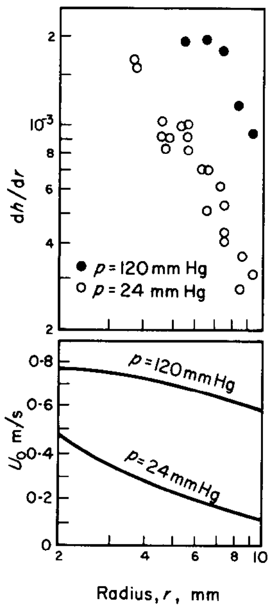


FIG. 4.

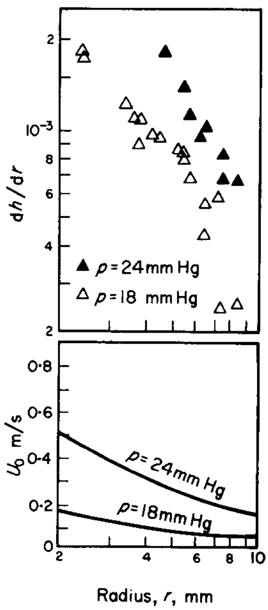


FIG. 5.

FIG. 4. Local gradient dh/dr and local spreading velocity of bubble U_0 in case of water. H = 0·25 mm.

FIG. 5. Local gradient dh/dr and local spreading velocity of bubble U_0 in case of methanol. H = 0·25 mm.

the center as shown in Fig. 2(a), its effect does not extend to the micro-liquid-layer in the region far from the center. This suggests that the formation of micro-liquid-layer can be practically regarded as a local phenomenon related to the movement of gas–liquid interface near the front meniscus of a growing bubble.

Secondly, it should be mentioned that increasing rate of the thickness of micro-liquid-layer with the radial distance from the center becomes greater as the spreading velocity of bubble, U_0, increases. This is a remarkable fact though it seems to be contrary to the conventional knowledge of hydrodynamics. Figures 4 and 5 show the experimental results as to the local gradient of the surface of liquid-layer, dh/dr (h = thickness of liquid layer, and r = radial coordinate), as well as the local spreading velocity of bubble, U_0, at the location of r. Figure 4 corresponds to the case of water with the initial pressure of air chamber of $p = 120$ and 24 mmHg; and Fig. 5 the case of methanol with $p = 24$ and 18 mmHg.

4.2 *Qualitative examination of phenomenon*

In order to bring the experimental results of Figs. 4 and 5 into a proper correlation, a simple qualitative inquiry will be attempted. Figure 6(a) shows the ordinary case of radial flow of fluid in a narrow space between a pair of parallel disks with pressure drop in the direction of flow, in which the pressure drop is balanced with viscous resistance of fluid. However, in the case of inquiry which is shown schematically in Fig. 6(b), viscosity of gas is so small compared with viscosity of liquid that the pressure within a spreading bubble may be regarded as approximately uniform. Consequently, if the effect of the surface tension is not taken into consideration, the field of flow may be roughly divided into two regions as shown in Fig. 6(b): the region A where the pressure is kept at a uniform value and the region B where the pressure drop exists. Liquid-layer which is exposed to gas is generally so thin that the pressure of gas is exerted directly on the liquid-layer, that is, the pressure within the liquid-layer is kept at a uniform value.

However, the region A must be connected with the region B in a continuous manner, hence the liquid in the region A ought to flow with a considerable velocity in the neighborhood of the region B at least (the gas–liquid interface is also moving here). In order to create the aforementioned flow in liquid resisting the viscous force of liquid, there must be variation of pressure within the liquid-layer falling in the direction of flow, and it should be noted that the variation of pressure cannot arise unless the surface tension operates on the gas–liquid interface.

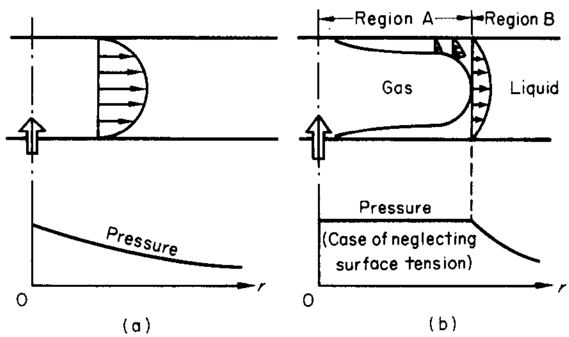


FIG. 6. Radial flow in a space between a pair of parallel disks.

As to the curvature of gas–liquid interface, it may be regarded as approximately zero for the stationary part of micro-liquid-layer which is extremely thin compared with its extension and is substantially in parallel with the solid wall, whereas the curvature increases abruptly near the front meniscus of bubble. Consequently, it may be presumed for the latter region that the surface tension gives rise to sufficient pressure drop within the liquid-layer by which the liquid is driven to flow. Strictly speaking, the curvature in the stationary part of micro-liquid-layer does not vanish however small it may be, so that the stationary layer must have variation of pressure due to the surface tension also. However, the variation of pressure is extremely small and besides, a micro-liquid-layer with thickness of the order of microns cannot create a measurable flow owing to the great resistance of viscosity, so that the micro-liquid-layer remains substantially in a stationary state. It may also be clear from the aforementioned that the shearing stress of gas stream exerted on the liquid surface can be hardly a controlling factor for the phenomenon concerned.

4.3 *Correlation of experimental results*

In compliance with the matters mentioned in Sections 4.1 and 4.2, it may be assumed that the physical quantities which have dominant effects on the local gradient, $\mathrm{d}h/\mathrm{d}r$, of the surface of stationary micro-liquid-layer are the local spreading velocity of bubble, U_0; the viscosity of liquid, μ; the surface tension, σ; the radial distance, r; and the normal distance between a pair of parallel disks, $2H$ (half of normal distance is represented by H). Although a stationary micro-liquid-layer extends its domains continuously through an unsteady phenomenon, the velocity of liquid ought to be very small near the position where the stationary liquid-layer is being made freshly, so that the effect of the force of inertia may be neglected. The effect of gravity also may be neglected.

Dimensionless correlation of physical quantities is accordingly written in the simplest form as follows:

$$\frac{\mathrm{d}h}{\mathrm{d}r} = C\left(\frac{\mu U_0}{\sigma}\right)^m \left(\frac{H}{r}\right)^n \tag{1}$$

where C, m and n are a numerical constant respectively. Then, if equation (1) is examined carefully with the experimental results given in Figs. 4 and 5, it yields $C = 0{\cdot}8$, $m = \frac{2}{3}$ and $n = 1$ as the most appropriate value of the respective numerical constant. Consequently, equation (1) can be quantitatively fixed as

$$\frac{\mathrm{d}h}{\mathrm{d}r} = 0{\cdot}8\left(\frac{\mu U_0}{\sigma}\right)^{\frac{2}{3}} \frac{H}{r} \tag{2}$$

whose comparison with the experimental results is shown in Fig. 7. Experimental data lie scattered to some extent in Fig. 7, but it is

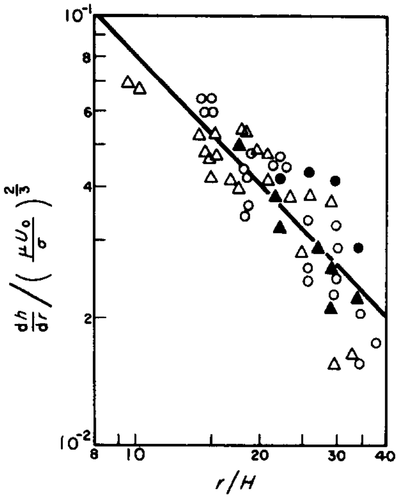


FIG. 7. Correlation of experimental results.

presumed that the scattering is mostly due to the difficulty of the present study submitting to not only various disturbances which are sometimes induced to a micro-liquid-layer but also measurement of such local values as $\mathrm{d}h/\mathrm{d}r$ and U_0. Here, note that water is nearly 1·7 times in viscosity, μ, and nearly 3·2 times in surface tension, σ, as great as methanol at the normal temperature.

5. THEORETICAL DISCUSSION

5.1 *Fundamental equations*

A liquid-layer flowing radially along a solid plane is shown in Fig. 8, where r represents the radial coordinate; y the vertical coordinate; h the thickness of liquid-layer at the location of r;

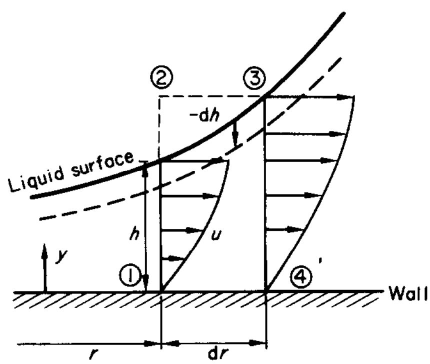


FIG. 8. Radial flow of liquid along a solid plane.

and u the horizontal component of velocity of liquid at any height, y, in the liquid-layer. Volumetric flow rate of liquid through a control surface ○ ③, which is a cylindrical surface, is given by $2\pi rV$ where $V \equiv \int_0^h u\,\mathrm{d}y$.

(i) *Equation of continuity.* Difference between the mass of liquid flowing into and out of a control volume ① ② ③ ④ should be balanced with the change of height of surface of liquid-layer, so that the following equation of continuity is immediately derived:

$$\frac{1}{r}\frac{\partial(rV)}{\partial r} = -\frac{\partial h}{\partial t} \tag{3}$$

where t is the time.

(ii) *Equation of motion.* In compliance with Sections 4.2 and 4.3, the gas in contact with the surface of liquid-layer is assumed to be kept at a uniform pressure, and the force of inertia as well as the gravity are neglected. Since the liquid-layer is so thin that the pressure originated by the surface tension can be assumed to extend immediately across a section of $y = 0$–h. Then, the change of curvature along the surface of liquid originates pressure gradient within the liquid-layer, with which the viscous resistance pertaining to flow is balanced, leading to

$$\mu\frac{\partial^2 u}{\partial y^2} = -\sigma\frac{\partial \rho}{\partial r}$$

where μ is the coefficient of viscosity of liquid, σ the surface tension, and ρ the curvature of liquid surface (ρ is negative when the center of curvature is in the side of liquid). Integrating the aforementioned equation with respect to y under the boundary conditions that $u = 0$ at $y = 0$ and $\partial u/\partial y = 0$ at $y = h$, it yields

$$u = \frac{\sigma}{\mu}\frac{\partial \rho}{\partial r}y\left(h - \frac{y}{2}\right)$$

Substitution of u thus obtained into $V \equiv \int_0^h u\,\mathrm{d}y$ and integration yield

$$V = \frac{\sigma}{3\mu}\frac{\partial \rho}{\partial r}h^3 \tag{4}$$

which is regarded as the equation of motion for the present case.

(iii) *Equation of liquid surface.* As shown with a broken curve in Fig. 8, the surface of liquid layer is assumed to descend by an infinitesimal quantity of $\mathrm{d}h$ in an infinitesimal interval of $\mathrm{d}t$. This normal displacement of liquid surface can be observed as a horizontal displacement of liquid surface also. In the latter case, if the horizontal velocity of moving surface of liquid is U, horizontal displacement in an interval of $\mathrm{d}t$ is $U\,\mathrm{d}t$. The two displacements aforementioned, $\mathrm{d}h$ and $U\,\mathrm{d}t$, are not independent of each other but related together with the gradient of liquid surface, $\partial h/\partial r$, by the following geometrical relation:

$$\frac{U\,\mathrm{d}t}{-\mathrm{d}h} = \frac{\partial h}{\partial r}, \quad \text{or} \quad U = -\frac{\partial h}{\partial t}\Big/\frac{\partial h}{\partial r}. \tag{5}$$

It should be noticed that equation (5) has no relation to equation (3) concerned with the conservation of mass. Besides, U is a quantity different from the velocity of liquid, u, at the liquid surface of $y = h$, and the two velocities of U and $u_{y=h}$ do not take the same value in general. For instance, if there is no difference between the mass of liquid flowing into and out of a control volume ① ② ○ ④ in Fig. 8, the liquid surface remains stationary, that is, $U = 0$ in spite of $u_{y=h} > 0$.

5.2 *Theoretical analysis*

If a differential equation, which can be derived by eliminating V from equations (3) and (4), is solved with the proper boundary and initial conditions, a solution of $h(r, t)$ may be obtained. Then, equation (5) gives $U(r, t)$ immediately, and the relation between h and U is obtained thereby. However, this way of analysis is very difficult.

Meanwhile, quite simple features have already been observed in the actual phenomenon as mentioned in Section 4.1. Therefore, let us simplify the analysis taking the empirical features of the phenomenon into consideration and examine the theoretical background for the existence of an experimental equation (2).

According to Section 4.1, the gas–liquid interface near the front meniscus of bubble moves rapidly, while the micro-liquid-layer which is formed continuously behind the moving part remains stationary. Approximately speaking, the rapidly moving interface may be assumed to move for the most part with a uniform velocity of U_0. Between the moving region of $U = U_0$ and the stationary region of $U = 0$, there should be an intermediate region where U varies from U_0 to zero. Although this intermediate region is of great importance participating to the formation of micro-liquid-layer, the extension of the region itself is presumed to be very confined. It is not necessarily useless, therefore, for the study of the local profile of micro-liquid-layer to examine the gradient of the interface moving with a uniform velocity of $U = U_0$ near the solid wall.

Substituting equations (3) and (4) into equation (5) and putting $U = U_0$, it yields

$$\left(\frac{3\mu U_0}{\sigma} - 3h^2\frac{d\rho}{dr}\right)\frac{dh}{dr} = h^3\left(\frac{d^2\rho}{dr^2} + \frac{1}{r}\frac{d\rho}{dr}\right) \quad (6)$$

which is a differential equation regarding the instantaneous profile of the interface moving with a uniform velocity of U_0.

Equation (6) includes $d\rho/dr$ and $d^2\rho/dr^2$ relating to the curvature of liquid-surface, ρ.

Since the liquid-surface shown in Fig. 8 is a surface of revolution, ρ can be written as $\rho = \rho_1 + \rho_2$ where ρ_1 is the sectional curvature on the plane of paper, and ρ_2 the sectional curvature on the plane orthogonal to the plane of paper. For a flattened bubble of r in radius held between a pair of parallel disks, variations of the two sectional curvatures along the interface are in the order of $\rho_1 \doteqdot 0$–$1/H$ (section of front meniscus is nearly semicircle with radius of H) and $\rho_2 \doteqdot 0$–$1/r$ respectively. However, the present study is particularly concerned with the region of $r/H \gg 1$ (see the abscissa of Fig. 7), where the variation of ρ_1 is much greater than that of ρ_2. In addition, it is presumably certain that the greater part of the variation of $\rho_1 \doteqdot 0$–$1/H$ concentrates on the part of liquid surface near the solid wall. Accordingly, it is permissible to assume $\rho = \rho_1$ approximately neglecting the effect of ρ_2 as compared with that of ρ_1. As is well known in mathematics, ρ_1 is generally given by

$$\rho_1 = \frac{d^2h}{dr^2}\Big/\left[1 + \left(\frac{dh}{dr}^2\right)\right]^{\frac{3}{2}}.$$

In the present case, however, the neighborhood of the solid wall is dealt with particularly and the gradient of liquid surface is not great there, so that $dh/dr \ll 1$ may be assumed to give

$$\rho = \rho_1 = d^2h/dr^2.$$

Substituting this into equation (6) and adopting dimensionless variables of ζ and η defined by

$$h \equiv \left(\frac{3\mu U_0}{\sigma}\right)^{\frac{1}{3}} H\zeta, \quad \text{and} \quad r \equiv H\eta \quad (7)$$

it yields

$$\left(1 - 3\zeta^2\frac{d^3\zeta}{d\eta^3}\right)\frac{d\zeta}{d\eta} = \zeta^3\left(\frac{d^4\zeta}{d\eta^4} + \frac{1}{\eta}\frac{d^3\zeta}{d\eta^3}\right). \quad (8)$$

Instantaneous profile of the moving interface near the solid wall, $\zeta(\eta)$, is to be obtained by solving equation (8), It is, however, quite impossible because instantaneous boundary conditions cannot be known. What we do now is to examine tentatively the value of $d\zeta/d\eta$ (gradient of interface) which may be obtained immediately from equation (8). Anyhow, equation (8) has a complicated form so that an approximate simplification will be attempted in the analysis. In the first place, let us assume that the following relation holds:

$$\frac{d^3\zeta}{d\eta^3} = \frac{K}{\zeta^n} \quad (9)$$

where K and n are a numerical constant respectively. Differentiation of equation (9) with respect to η yields

$$\frac{d^4\zeta}{d\eta^4} = -n\frac{K}{\zeta^{n+1}}\frac{d\zeta}{d\eta}. \quad (10)$$

Then, substitution of both equations (9) and (10) into equation (8) leads to

$$\frac{d\zeta}{d\eta}\left\{1 + (n-3)\frac{K}{\zeta^{n-2}}\right\} = \frac{K}{\eta}\frac{1}{\zeta^{n-3}}.$$

If $n = 3$ is assumed additionary here, $d\zeta/d\eta$ reduces to a function of η only, that is

$$\frac{d\zeta}{d\eta} = \frac{K}{\eta}. \tag{11}$$

If equation (11) is consistent with the preceding assumptions, equation (9) with $n = 3$, it is obvious that equation (11) is equivalent to equation (8).

Since equation (9) has a particular solution of $\zeta = \sqrt[4]{(64K/15)}\ \eta^{\frac{3}{4}}$ when $n = 3$, there is a case in which equation (9) can be replaced by $d^3\zeta/d\eta^3 = K(15/64K)^{\frac{3}{4}}\ \eta^{2\cdot25}$. On the other hand, $d^3\zeta/d\eta^3 = 2K/\eta^3$ is derived from equation (11). Unfortunately, the exponents of η, 2·25 and 3, in the two expressions of $d^3\zeta/d\eta^3$ fail to have a complete coincidence, but they are in proximity. Therefore, if the variational range of η, namely of r/H, is confined within a limited interval, it is possible to keep an approximate correspondence between equations (9) and (11). In other words, there is a case in which equation (8) can be roughly replaced by equation (11). If equation (11) is reversely transformed with equation (7), we obtain

$$\frac{dh}{dr} = K\left(\frac{3\mu U_0}{\sigma}\right)^{\frac{1}{3}}\frac{H}{r} \tag{12}$$

which is concerned with the instantaneous gradient of an interface moving with a uniform velocity of U_0 near the solid wall.

Equation (2) obtained experimentally in Section 4.3 is concerned with the local gradient of the micro-liquid-layer at the stationary state of $U = 0$. However, it should be remembered that the place where the stationary micro-liquid-layer is being made freshly, is moving with the velocity of $U = U_0$. In other words, equation (2) also gives the local gradient of the stationary micro-liquid-layer at the front part which is extending with the velocity of $U = U_0$. Therefore, if there is a kind of intimate relation between the gradient of the stationary micro-liquid-layer at the front part and the gradient of the interface moving with the velocity of $U = U_0$ near the solid wall, it is not so strange. The fact, that the theoretical equation (12) has a form similar to the experimental equation (2), may be accordingly regarded as a theoretical support to the experimental result.

5.3 *Additional remarks*

In principle, complete coincidence of equations (2) and (12) should not be expected, and it is rather reasonable that the two equations differ in the exponent of the dimensionless term of $\mu U_0/\sigma$. In this connection, a tentative discussion will be stated below. The intermediate region between the stationary interface of $U = 0$ and the moving interface of $U = U_0$, where U varies abruptly, participates really in the formation of micro-liquid-layer. If it is assumed for sake of simplicity that the interface in the intermediate region moves with an average velocity of αU_0 where $\alpha < 1$, the analysis in the preceding section can be applied without any alteration except the replacement of U_0 by αU in equation (12). The ratio, α, of the average velocity to the boundary velocity should naturally depend on the state of flow. Remembering that the phenomenon concerned is dominated by the dimensionless number of $\mu U_0/\sigma$, it is permissible to assume that α is a function of $\mu U_0/\sigma$. Accordingly, if α is assumed tentatively to be proportional to $(\mu U_0/\sigma)^k$ where k is a numerical constant, equation (12) gives immediately

$$\frac{dh}{dr} = \text{const.}\left(\frac{\mu U_0}{\sigma}\right)^{(1+k)/3}\frac{H}{r}. \tag{13}$$

This may be regarded as an intermediate equation between equations (2) and (12).

6. COMPARISON WITH NUCLEATE BOILING BETWEEN A PAIR OF PARALLEL DISCS

In the experimental study of Katto and Yokoya [8], which has been mentioned in

Chapter 1, boiling of water at the atmospheric pressure was dealt with, and an isolated, flattened vapor bubble was generated in the space between a pair of parallel disks of 10 mm dia. heating the lower disk. Normal distance of the space was 0·4 mm, namely $2H = 0{\cdot}4$ mm. Local spreading velocity of bubble, U_0, which varied with the location of r, was measured and formularized for the range of $r \gg H$ ($r = 2{\cdot}5$–5 mm) as follows:

$$U_0 = 107\,000\sqrt{r}\ \text{m/h} = 29{\cdot}8\sqrt{r}\ \text{m/s} \quad (14)$$

where r is in metres. Distribution of the thickness of the micro-liquid-layer formed by the vapor bubble was estimated through local drying velocity of the liquid-layer at any position, yielding $h = 1{\cdot}58 \times 10^{-4}\sqrt{r}$ where h and r are in metres. Therefore, the local gradient of liquid surface in the present case of boiling is given immediately as follows:

$$\frac{\mathrm{d}h}{\mathrm{d}r} = 0{\cdot}79 \times 10^{-4}\frac{1}{r^{0{\cdot}5}}. \quad (15)$$

The local spreading velocity of vapor bubble, U_0, increases with r in the present case as shown in equation (14) owing to the increment of the surface of micro-liquid-layer on which evaporation takes place supplying the vapor to the growing bubble. On the other hand, U_0 decreases with r for the air bubble which grows receiving air from an air chamber as shown in Figs. 4 and 5. In any case, however, equation (2) should presumably hold so long as the local spreading velocity, U_0, is used. Substituting U_0 of equation (14) into equation (2), and adopting physical properties which correspond to the experimental condition of boiling of water at the atmospheric pressure ($\mu = 0{\cdot}290 \times 10^{-4}$ kg s/m², $\sigma = 6{\cdot}00 \times 10^{-3}$ kg/m, $H = 2 \times 10^{-4}$ m), it gives

$$\frac{\mathrm{d}h}{\mathrm{d}r} = 0{\cdot}44 \times 10^{-4}\frac{1}{r^{0{\cdot}66}} \quad (16)$$

where r is in metres.

It may be noticed that equation (16) is near akin to equation (15) derived immediately from the measurement of thickness of micro-liquid-layer. In this connection, Table 1 shows a comparison of the value of $\mathrm{d}h/\mathrm{d}r$ between the two equations (15) and (16) within the range of $r = 2{\cdot}5$–$5{\cdot}0$ mm. It is seen that equation (15) gives smaller values of $\mathrm{d}h/\mathrm{d}r$ by about 25–30 per cent compared with equation (16), but this discrepancy may be attributed mainly to the underestimation of h in the experiment of boiling. In fact, Katto and Yokoya [8] have already mentioned the discrepancy of the same origin in connection with the comparison of the analytical and observed values of U_0. The underestimation of h may be attributed to the error in the measurement of temperature of heated surface (the temperature exerts an immediate effect on the estimation of h).

Table 1. Comparison of values of $\mathrm{d}h/\mathrm{d}r$

r (mm)	2·5	3·0	4·0	5·0
$\mathrm{d}h/\mathrm{d}r$, equation (15)	$1{\cdot}58 \times 10^{-3}$	$1{\cdot}44 \times 10^{-3}$	$1{\cdot}25 \times 10^{-3}$	$1{\cdot}11 \times 10^{-3}$
$\mathrm{d}h/\mathrm{d}r$, equation (16)	$2{\cdot}29 \times 10^{-3}$	$2{\cdot}04 \times 10^{-3}$	$1{\cdot}74 \times 10^{-3}$	$1{\cdot}49 \times 10^{-3}$
$\dfrac{\mathrm{d}h/\mathrm{d}r\text{, equation (15)}}{\mathrm{d}h/\mathrm{d}r\text{, equation (16)}}$	0·69	0·71	0·72	0·75

Anyhow, summarizing the aforementioned, it may be concluded that equation (2) is considerably reliable even when it is applied to the nucleate boiling in the narrow space.

7. CONCLUSIONS

(1) A flattened air bubble has been generated artificially by air-injection in a narrow space between a pair of parallel disks filled with a liquid, and the thickness of the micro-liquid-layer formed at the base of the bubble has been measured by optical means.

(2) The gas–liquid interface can be approximately divided into two discrete regions: the moving region near the front meniscus of bubble and the stationary region.

(3) The formation of micro-liquid-layer can be practically regarded as a local phenomenon related to the movement of gas–liquid interface near the front mensicus, and local profile of micro-liquid-layer is given by the following experimental equation within the region of $r/H \gg 1$:

$$\frac{\mathrm{d}h}{\mathrm{d}r} = 0{\cdot}8\left(\frac{\mu U_0}{\sigma}\right)^{\frac{2}{3}}\frac{H}{r}.$$

(4) The aforementioned equation is not only supported by a theoretical analysis, but also reliable even when it is applied to the micro-liquid-layer observed in an experiment of nucleate boiling in a narrow space between a pair of parallel disks.

(5) It is presumed generally that the formation of micro-liquid-layer on a solid surface with a growing bubble is a phenomenon related to the flow of viscous liquid-layer being driven with the pressure drop generated from the surface tension and the change of curvature of liquid surface.

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MÉCANISME DU PRINCIPE DE LA FORMATION D'UNE MICROCOUCHE LIQUIDE SUR UNE SURFACE SOLIDE AVEC UNE BULLE QUI CROÎT PENDANT L'ÉBULLITION NUCLÉÉE

Résumé—On étudie en rapport avec l'ébullition nucléée le mécanisme de la formation d'une microcouche liquide entre une surface solide et une balle qui croît. Afin d'approcher la nature essentielle du sujet, une bulle d'air aplatie est produite dans un espace étroit compris entre une paire de disques parallèles et rempli par un liquide, et le comportement de la couche liquide formée sur la surface solide est observé par des moyens optiques.

En accord avec les résultats expérimentaux, la formation de la microcouche liquide est regardée comme un phénomène local lié à l'écoulement du liquide près du ménisque frontal de la bulle; et une équation empirique est obtenue eu égard au profil de la microcouche liquide.

Une analyse théorique est entreprise afin d'étudier l'écoulement de la couche liquide dans lequel la tension superficielle joue un rôle important. Elle fournit une base théorique pour l'équation empiriaue mentionée ci-dessus. Les résultats expérimentaux de la microcouche liquide obtenus dans l'ébullition nucléée de l'eau dans un espace étroit compris entre une paire de disques parallèles sont également employés pour examiner la fiabilité de l'équation empirique.

En résumé, on en conclut qu'une couche liquide placée entre une bulle et une surface solide est sujette à une variation spatiale de pression due à l'effet de la tension superficielle, et à cause de cela la couche liquide est obligée de s'écouler contre la force visqueuse, en laissant derrière ell une microcouche liquide d'une épaisseur de l'ordre de quelques microns.

GRUNDSÄTZLICHER MECHANISMUS DER MIKRO-FLÜSSIGKEITSSCHICHTBILDUNG AN EINER FESTEN OBERFLÄCHE MIT EINER WACHSENDEN BLASE BEI DER BLASENVERDAMPFUNG

Zusammenfassung—Es wird der Mechanismus der Mikro-Flüssigkeitsschichtbildung zwischen einer festen Oberfläche und einer wachsenden Blase bei der Blasenverdampfung untersucht. Um dem wesentlichen Charakter des Themas nahezukommen, wurde zwischen zwei parallelen kreisrunden Scheiben in einem engen mit Flüssigkeit gefüllten Spalt eine flachgedrückte Blase erzeugt und das Verhalten der Flüssigkeitsschicht mit optischen Mitteln beobachtet.

Entsprechend den experimentellen Ergebnissen wird die Bildung der Mikro-Flüssigkeitsschicht nahezu als örtliches Phenomen betrachtet, das in Beziehung zur Flüssigkeitsströmung in der Nähe der Vorderseite des Blasenmeniskus steht. Eine empirische Beziehung unter Berücksichtigung des Profils der Mikro-Flüssigkeitsschicht wird abgeleitet.

Mit einer theoretischen Analyse wird versucht, die Strömung der Flüssigkeitsschicht zu untersuchen, in der die Oberflächenspannung eine wichtige Rolle spielt.

Sie bildet die theoretische Grundlage für die oben erwähnte empirische Gleichung. Experimentelle Ergebnisse der Mikro-Flüssigkeitsschichtbildung bei der Blasenverdampfung von Wasser in einem engen Spalt zwischen zwei Scheiben werden ferner verwendet um die Zuverlässigkeit der empirischen Gleichung zu prüfen.

Zusammenfassend kann man sagen, dass eine zwischen einer Blase und der festen Oberfläche befindliche Flüssigkeitsschicht der Grund für die Lageänderung des Druckes unter dem Einfluss der Oberflächenspannung ist, wobei die Flüssigkeitsschicht gezwungen wird, den Zähigkeitskräften entgegen zu strömen, indem sie eine Mikro-Flüssigkeitsschicht mit einer Dicke in der Grössenordnung von Mikrons hinter sich lässt.

МЕХАНИЗМ ОБРАЗОВАНИЯ ЖИДКОГО МИКРОСЛОЯ МЕЖДУ ТВЕРДОЙ ПОВЕРХНОСТЬЮ И РАСТУЩИМ ПУЗЫРЬКОМ ПРИ ПУЗЫРЬКОВОМ КИПЕНИИ

Аннотация—Исследуется механизм образования жидкого микрослоя между твёрдой поверхностью и растущим пузырьком при пузырьковом кипении. Для того, чтобы рассмотреть истинную природу предмета, создавался плоский пузырёк воздуха в заполненной жидкостью узкой щели между двумя параллельными дисками, и поведение жидкого слоя, образованного на твёрдой поверхности, наблюдалось с помощью оптических средств.

Согласно экспериментальным результатам образование жидкого микрослоя рассматривается как местное явление, связанное с течением жидкости у переднего мениска пузыря. Выведено эмпирическое уравнение для профиля жидкого микрослоя.

Делается попытка применить теоретический анализ для изучения течения жидкого слоя, в котором важную роль играет поверхностное натяжение. Он даёт теоретическую основу для вышеназанного эмпирического уравнения. Экспериментальные данные о жидком микрослое, полученные при пузырьковом кипении воды в узком иространстве между двумя параллельными дисками, используются также для исследования надёжности эмпирического уравнения.

Обобщая данные исследования, сделан вывод о том, что давление жидкого слоя, заключённого между пузырьком и твёрдой поверхностью, претерпевает позиционные изменения под влиянием поверхностного натяжения, тем самым заставляя жидкий слой течь против сил вязкости, оставляя позади жидкий микрослой толщиной порядка нескольких микронов.